

How do I Implement SPC for Short Production Runs (Part II)?



Steven Wachs
Principal Statistician
Integral Concepts, Inc.

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In Part I of this article, we introduced the concept of utilizing *Deviation from Nominal (DNOM)* control charts for short production runs. These charts allow us to monitor process characteristics over time even when the units being controlled have varying nominal values. DNOM charts assume that the process variability (i.e. standard deviation) does not vary significantly by part type. However, often this assumption does not hold. Characteristics with larger nominal values tend to have more variation than characteristics with smaller nominal values. In Part II we discuss how to test whether or not significant differences in variability exist and if so, how to modify the DNOM methods and charts to handle this situation.

Testing for Variation Differences

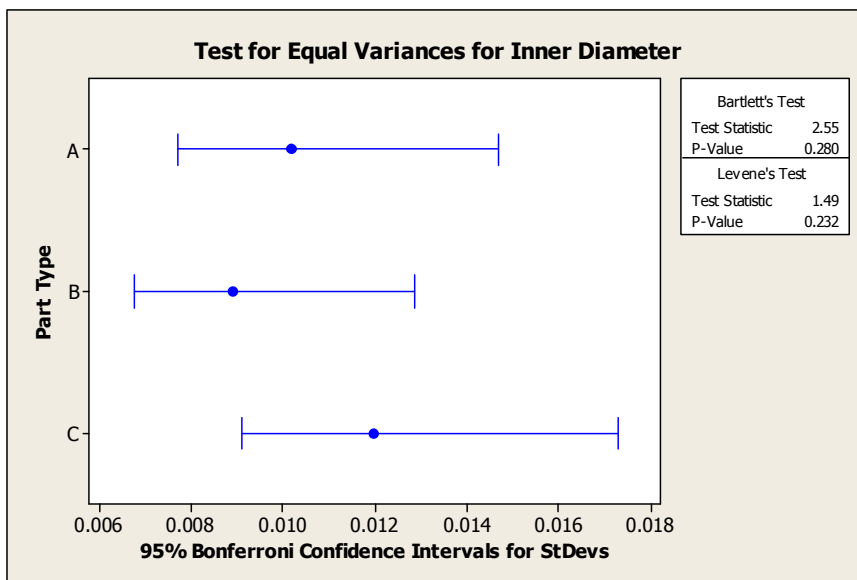
A statistical test may be performed to test whether the differences observed in the standard deviations (or variances) of multiple groups represent a statistically significant difference. Statistically significant differences mean that it is very unlikely that the magnitude of the differences occurred by chance.

Recall the data we presented in the example in Part I of the article:

Sample	Part Type	Nominal	Measurements			Deviation from Nominal			Chart Statistics		
			m ₁	m ₂	m ₃	x ₁	x ₂	x ₃	xbar	R	S
1	A	17.400	17.408	17.404	17.407	0.008	0.004	0.007	0.0062	0.003	0.0016
2	A	17.400	17.403	17.404	17.414	0.003	0.004	0.014	0.0068	0.011	0.0063
3	A	17.400	17.392	17.393	17.375	-0.008	-0.007	-0.025	-0.0135	0.018	0.0101
4	A	17.400	17.387	17.400	17.387	-0.013	0.000	-0.013	-0.0090	0.013	0.0074
5	A	17.400	17.400	17.399	17.420	0.000	-0.001	0.020	0.0063	0.022	0.0123
6	A	17.400	17.408	17.400	17.399	0.008	0.000	-0.001	0.0022	0.010	0.0053
7	A	17.400	17.409	17.396	17.395	0.009	-0.004	-0.005	-0.0001	0.014	0.0075
8	A	17.400	17.411	17.418	17.402	0.011	0.018	0.002	0.0102	0.016	0.0082
9	A	17.400	17.397	17.403	17.387	-0.003	0.003	-0.013	-0.0043	0.015	0.0078
10	A	17.400	17.415	17.400	17.389	0.015	0.000	-0.011	0.0014	0.026	0.0130
11	B	12.700	12.698	12.702	12.686	-0.002	0.002	-0.014	-0.0046	0.016	0.0084
12	B	12.700	12.697	12.704	12.687	-0.003	0.004	-0.013	-0.0036	0.017	0.0086
13	B	12.700	12.698	12.713	12.708	-0.002	0.013	0.008	0.0062	0.016	0.0081
14	B	12.700	12.720	12.687	12.698	0.020	-0.013	-0.002	0.0014	0.033	0.0170
15	B	12.700	12.696	12.702	12.707	-0.004	0.002	0.007	0.0017	0.011	0.0056
16	B	12.700	12.709	12.702	12.702	0.009	0.002	0.002	0.0044	0.007	0.0039
17	B	12.700	12.695	12.686	12.712	-0.005	-0.014	0.012	-0.0022	0.026	0.0133
18	B	12.700	12.695	12.691	12.696	-0.005	-0.009	-0.004	-0.0061	0.005	0.0028
19	B	12.700	12.685	12.693	12.695	-0.015	-0.007	-0.005	-0.0090	0.009	0.0049
20	B	12.700	12.712	12.695	12.700	0.012	-0.005	0.000	0.0024	0.016	0.0085
21	C	10.500	10.487	10.499	10.494	-0.013	-0.001	-0.006	-0.0067	0.012	0.0060
22	C	10.500	10.502	10.508	10.496	0.002	0.008	-0.004	0.0023	0.012	0.0059
23	C	10.500	10.504	10.498	10.510	0.004	-0.002	0.010	0.0038	0.012	0.0060
24	C	10.500	10.500	10.496	10.490	0.000	-0.004	-0.010	-0.0051	0.010	0.0050
25	C	10.500	10.485	10.481	10.485	-0.015	-0.019	-0.015	-0.0163	0.003	0.0020
26	C	10.500	10.526	10.516	10.494	0.026	0.016	-0.006	0.0121	0.032	0.0164
27	C	10.500	10.519	10.501	10.508	0.019	0.001	0.008	0.0093	0.018	0.0092
28	C	10.500	10.520	10.503	10.504	0.020	0.003	0.004	0.0090	0.017	0.0096
29	C	10.500	10.512	10.486	10.512	0.012	-0.014	0.012	0.0030	0.026	0.0152
30	C	10.500	10.494	10.508	10.479	-0.006	0.008	-0.021	-0.0064	0.028	0.0141
									0.0001	0.0159	0.0083
									xbarbar	rbar	sbar

When only two groups are being tested, an F test is performed (if each group follows a normal distribution) or a Levene's Test is performed (if the normality assumption does not hold).

When more than two groups are being tested, a Bartlett's test is performed (if the groups follow a normal distribution) or a Levene's Test is performed (if the normality assumption does not hold).



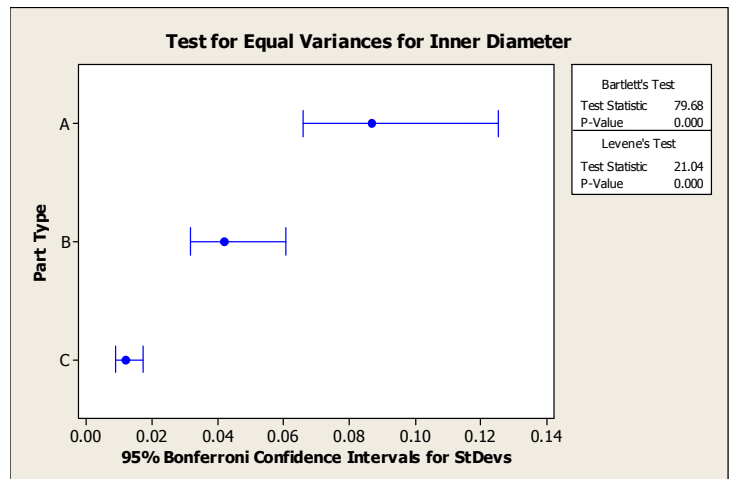
The above graphic illustrates the results of the hypothesis test for equal variances for the 3 part types shown in the previous data. The relatively large p values (> 0.05) indicate that we do not have sufficient evidence to conclude that the variances are different (with 95% confidence).

In this case, a regular DNOM chart (and S chart) would be appropriate.

Variation Differences

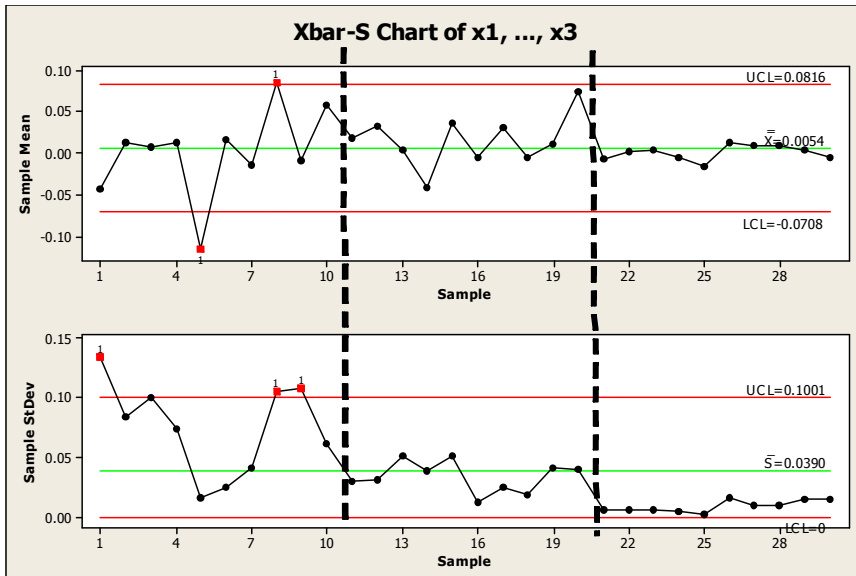
Now, we consider the case where the groups do not share a common degree of variation. Data on Inner Diameter on 3 Part Types is shown below. A statistical test is performed to test for equality of variances.

Sample	Part Type	Nominal (T)	Measurements		
			x_1	x_2	x_3
1	A	17.400	17.463	17.399	17.207
2	A	17.400	17.507	17.353	17.377
3	A	17.400	17.396	17.312	17.511
4	A	17.400	17.336	17.483	17.419
5	A	17.400	17.274	17.302	17.276
6	A	17.400	17.430	17.431	17.389
7	A	17.400	17.424	17.343	17.389
8	A	17.400	17.437	17.605	17.412
9	A	17.400	17.514	17.321	17.336
10	A	17.400	17.392	17.469	17.513
11	B	12.700	12.736	12.734	12.684
12	B	12.700	12.723	12.708	12.766
13	B	12.700	12.659	12.693	12.759
14	B	12.700	12.625	12.701	12.649
15	B	12.700	12.754	12.678	12.774
16	B	12.700	12.693	12.684	12.707
17	B	12.700	12.704	12.750	12.739
18	B	12.700	12.712	12.675	12.696
19	B	12.700	12.685	12.689	12.758
20	B	12.700	12.811	12.732	12.776
21	C	10.500	10.487	10.499	10.494
22	C	10.500	10.502	10.508	10.496
23	C	10.500	10.504	10.498	10.510
24	C	10.500	10.500	10.496	10.490
25	C	10.500	10.485	10.481	10.485
26	C	10.500	10.526	10.516	10.494
27	C	10.500	10.519	10.501	10.508
28	C	10.500	10.520	10.503	10.504
29	C	10.500	10.512	10.486	10.512
30	C	10.500	10.494	10.508	10.479



Clearly more variation exists for the larger diameter parts than for the smaller diameter parts. The very small p-values (< 0.0005) indicate that the differences in group variances are statistically significant. You can also see this by noticing that the estimated standard deviations of each group fall outside the confidence intervals for the other groups' estimated standard deviations.

What if we hadn't checked and just constructed a regular DNOM and S chart? Can the 3 part types (segregated by the dashed vertical lines) be adequately monitored on these charts (on the next page)?



Standardized DNOM and Standardized S Charts

Since the control limits are based on the observed variation, we need a method to account for the variation differences between the groups. The solution to the problem is a Standardized DNOM chart (to monitor location) and Standardized S chart (to monitor variation).

The Standardized DNOM chart plots the *number of standard deviations* each sample average is from the nominal value. This is found simply by computing the deviation from nominal (as before) and dividing it by the estimated standard deviation *for that part type*. Since each plotted sample statistic accounts for its specific group standard deviation, the data can be combined onto one chart. The Standardized S Chart plots the ratio of the Sample Standard Deviation to the Group Standard Deviation. The following table illustrates the required calculations.

Sample	Part Type	Nominal (T_k)	Measurements			Chart Statistics			
			x_1	x_2	x_3	S_i	\bar{S}_k	$(\bar{X}_i - T_k)/\bar{S}_k$	S_i/\bar{S}_k
1	A	17.400	17.463	17.399	17.207	0.134		-0.587	1.794
2	A	17.400	17.507	17.353	17.377	0.083		0.167	1.115
3	A	17.400	17.396	17.312	17.511	0.100		0.084	1.340
4	A	17.400	17.336	17.483	17.419	0.074		0.172	0.992
5	A	17.400	17.274	17.302	17.276	0.016		-1.558	0.209
6	A	17.400	17.430	17.431	17.389	0.024		0.221	0.324
7	A	17.400	17.424	17.343	17.389	0.041		-0.199	0.545
8	A	17.400	17.437	17.605	17.412	0.105		1.133	1.409
9	A	17.400	17.514	17.321	17.336	0.108		-0.132	1.447
10	A	17.400	17.392	17.469	17.513	0.061	0.074	0.778	0.825
11	B	12.700	12.736	12.734	12.684	0.030		0.530	0.879
12	B	12.700	12.723	12.708	12.766	0.030		0.969	0.903
13	B	12.700	12.659	12.693	12.759	0.051		0.117	1.518
14	B	12.700	12.625	12.701	12.649	0.039		-1.236	1.151
15	B	12.700	12.754	12.678	12.774	0.051		1.054	1.511
16	B	12.700	12.693	12.684	12.707	0.012		-0.163	0.356
17	B	12.700	12.704	12.750	12.739	0.024		0.924	0.719
18	B	12.700	12.712	12.675	12.696	0.018		-0.168	0.542
19	B	12.700	12.685	12.689	12.758	0.041		0.310	1.230
20	B	12.700	12.811	12.732	12.776	0.040	0.034	2.173	1.189
21	C	10.500	10.487	10.499	10.494	0.006		-0.748	0.671
22	C	10.500	10.502	10.508	10.496	0.006		0.258	0.662
23	C	10.500	10.504	10.498	10.510	0.006		0.421	0.673
24	C	10.500	10.500	10.496	10.490	0.005		-0.570	0.556
25	C	10.500	10.485	10.481	10.485	0.002		-1.826	0.220
26	C	10.500	10.526	10.516	10.494	0.016		1.351	1.830
27	C	10.500	10.519	10.501	10.508	0.009		1.039	1.031
28	C	10.500	10.520	10.503	10.504	0.010		1.009	1.078
29	C	10.500	10.512	10.486	10.512	0.015		0.339	1.695
30	C	10.500	10.494	10.508	10.479	0.014	0.009	-0.720	1.583

The standardized DNOM is in the next to last column and the standardized S statistic is in the last column. The notation, key statistics, and control limits for both charts are shown below.

Standardized DNOM and Standardized S Notation and Control Chart Calculations

T_k = nominal (or target or average) value for k^{th} group

S_i = sample standard deviation (i^{th} sample)

\bar{S}_k = standard deviation of k^{th} group (average of group $S_{i's}$)

\bar{X}_i = sample average (i^{th} sample)

$$\text{Standardized DNOM} = \frac{(\bar{X}_i - T_k)}{\bar{S}_k}$$

$$\text{Standardized S} = \frac{S_i}{\bar{S}_k}$$

Standardized DNOM Chart

Center Line = 0

$$LCL = -A_3$$

$$UCL = +A_3$$

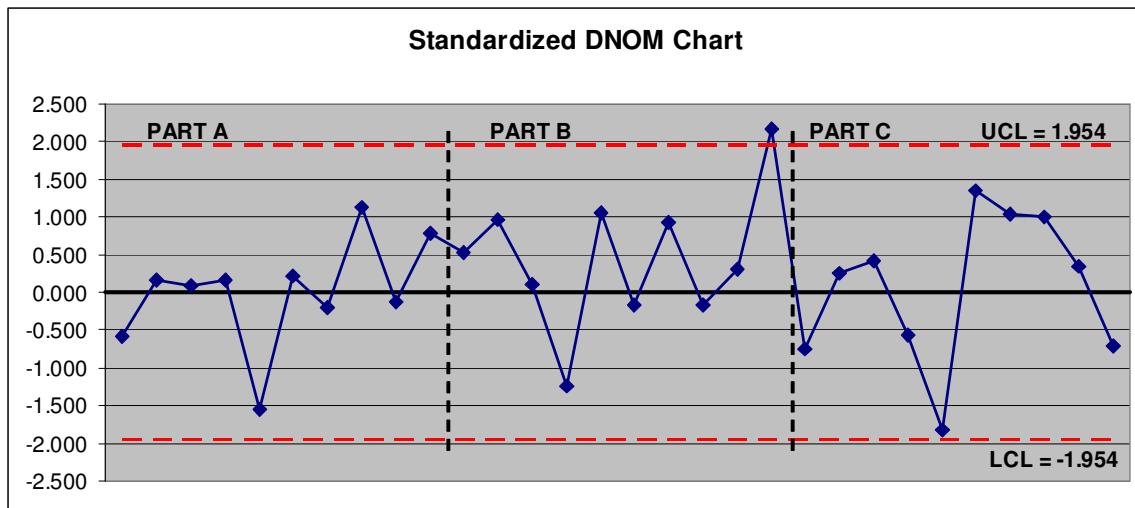
Standardized S Chart

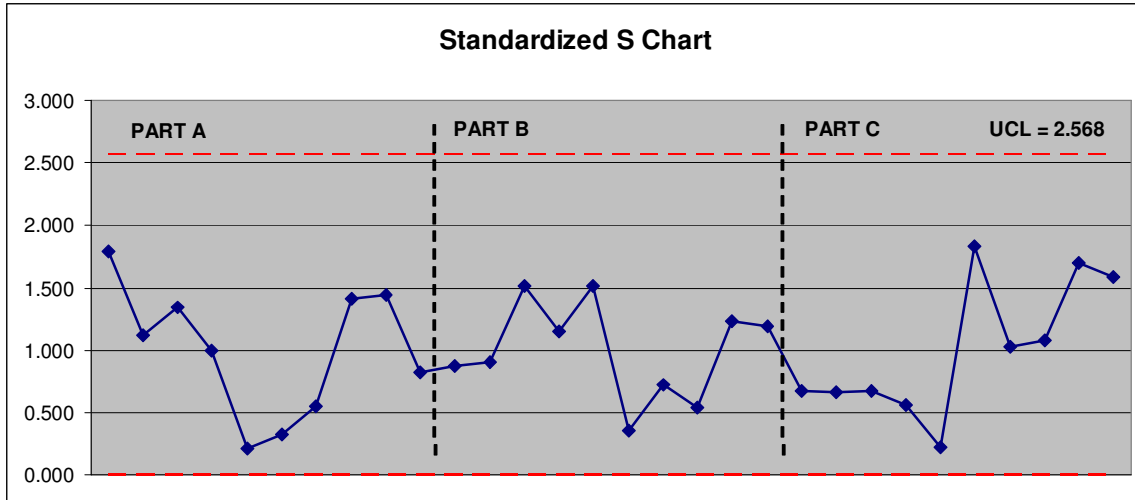
$$LCL = B_3$$

$$UCL = B_4$$

Table of Factors for Standardized DNOM & S			
n	A ₃	B ₃	B ₄
2	2.659	0	3.267
3	1.954	0	2.568
4	1.628	0	2.266
5	1.427	0	2.089
6	1.287	0.030	1.970
7	1.182	0.118	1.882
8	1.099	0.185	1.815
9	1.032	0.239	1.761
10	0.975	0.284	1.716

Standardized DNOM and Standardized S charts based on the above data and calculations are shown below.





Summary

This article completes the 2-part series describing control charts for short production runs. This article presented methods to test whether or not significant differences in variability among the part types exist. When significant differences in variability exist, Standardized DNOM and S charts should be utilized